

# Perfectly-Matched Bandstop Filters using Lossy Resonators

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**Abstract** — Normal realizations of bandstop resonators with finite unloaded  $Q$  suffer from degradation of performance due to dissipation loss. In this paper it is shown theoretically that there exists a class of second-order networks which simultaneously exhibit an ideal bandstop resonance, with infinite stopband attenuation, and a perfect match at all frequencies. Theoretical analysis is backed up with experimental results for three different physical realizations.

**Index Terms** — Passive filters, lossy circuits, notch filters.

## I. INTRODUCTION

Highly-selective filters are a necessity for many applications such as communications transceivers and radar systems. As filter selectivity is limited by the losses associated with the technology used to realize the filter, it is desirable to use design techniques which take the effects of losses into account. Such techniques include passive methods such as predistortion [1], and active approaches [2]. Active approaches suffer from issues such as their inherent nonlinearity which make their use for many applications limited [3]. The present work utilizes a passive approach to implement a narrowband bandstop filter with theoretically infinite stopband attenuation, while being perfectly matched in both the passband and stopband. This work builds upon the perfect-notch concept presented in [4] by showing that an identical transfer function can be realized with lossy allpass networks, which by nature are perfectly matched. Such networks could theoretically be placed in cascade to realize more advanced transfer functions. In this paper theory is presented which shows how a lossy allpass network can be used to realize a perfect notch, followed by a performance comparison to a conventional resonator. Prototype designs are presented, including novel dual-mode versions.

## II. THEORY

### A. Lossy allpass networks

Consider a symmetrical two-port network defined by even- and odd-mode admittances  $Y_e$  and  $Y_o$ . The S-parameters are then given by:

$$S_{11} = \frac{1 - Y_o Y_e}{(1 + Y_o)(1 + Y_e)} \quad (1) \quad S_{12} = \frac{Y_o - Y_e}{(1 + Y_o)(1 + Y_e)} \quad (2)$$

Now if  $Y_o = 1/Y_e$ , then  $S_{11} = 0$  for all  $\omega$ , the network is perfectly matched, and

$$S_{12} = \frac{1 - Y_e}{1 + Y_e}. \quad (3)$$

If the network is lossless then  $Y_e$  is a reactance function:

$$Y_e(j\omega) = j \frac{N(\omega)}{D(\omega)} \quad (4)$$

giving

$$S_{12}(j\omega) = \frac{D - jN}{D + jN} \quad (5)$$

and  $|S_{12}|^2 = 1$  for all  $\omega$  (an allpass network).

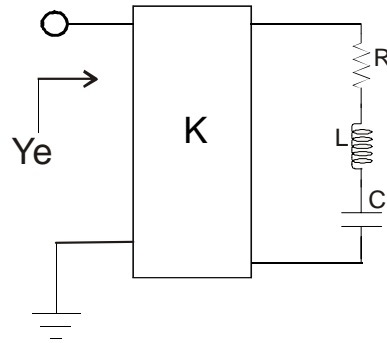


Fig. 1. Even-mode admittance of a lossy resonant circuit.

Now consider the case where  $Y_e$  is a lossy resonant circuit. For example, let

$$Y_e = Lp + \frac{1}{Cp} + R \quad (6)$$

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This admittance is given by the circuit shown in Fig. 1.

Assuming  $Y_o = 1/Y_e$  as before, then

$$|S_{12}(j\omega)|^2 = \frac{(1-R)^2 + (\omega L - 1/\omega C)^2}{(1+R)^2 + (\omega L - 1/\omega C)^2} \quad (7)$$

and if  $R = 1$ :

$$|S_{12}|^2 = \frac{1}{1 + \frac{4}{Q_u^2 \left[ \frac{\omega}{\omega_o} - \frac{\omega_o}{\omega} \right]^2}} \quad (8)$$

which is the transfer function of an ideal lossless bandstop resonator.  $Q_u$  is the unloaded  $Q$  which is equal to

$$Q_u = \frac{\omega_o L}{R} \quad (9)$$

and it may be shown that

$$\omega_2 - \omega_1 = \Delta_{3dB} = \frac{2\omega_o}{Q_u} \quad (10)$$

hence

$$\frac{\omega_o}{\Delta_{3dB}} = \frac{Q_u}{2} \quad (11)$$

The loaded  $Q$  of the resulting band-reject resonator is therefore half of the unloaded  $Q$  of the resonator. Shown in Fig. 2 is a practical matched notch circuit [4].

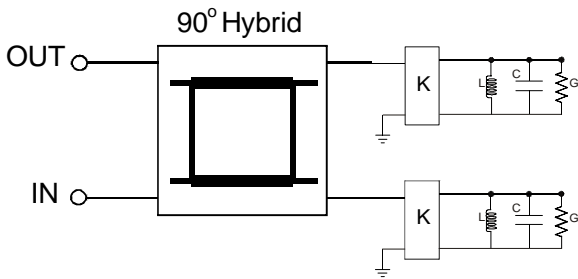


Fig. 2. Hybrid circuit implementation of a perfectly-matched notch filter.

### B. Comparison with conventional resonator

Consider a conventional bandstop resonator (Fig. 3a). The insertion loss is given by:

$$IL = 10 \log \left[ 1 + \frac{1+4R}{4R^2 + 4Q_u^2 R^2 \left[ \frac{\omega}{\omega_o} - \frac{\omega_o}{\omega} \right]^2} \right] dB \quad (12)$$

The 3-dB bandwidth is:

$$\Delta 3dB = \omega_o \sqrt{\frac{1+4R-4R^2}{4Q_u^2 R^2}} \quad (13)$$

Now let (11) equal to (13):

$$\frac{2}{Q_u} = \sqrt{\frac{1+4R-4R^2}{4Q_u^2 R^2}} \quad (14)$$

Therefore at  $\omega_o$ :

$$IL = 10 \log \left[ 1 + \frac{1+4R}{4R^2} \right] = 7.8 dB \quad (15)$$

That is, for the same unloaded  $Q$  and 3-dB bandwidth as a bandstop filter implemented using a lossy allpass network, the conventional notch resonator only gives 7.8 dB rejection.

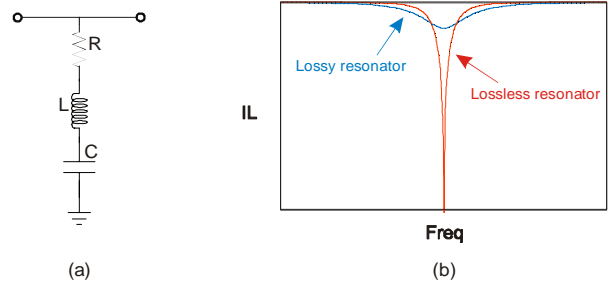


Fig. 3. (a) Conventional bandstop resonator. (b) Effect of finite  $Q$ .

### III. PROTOTYPE DESIGNS

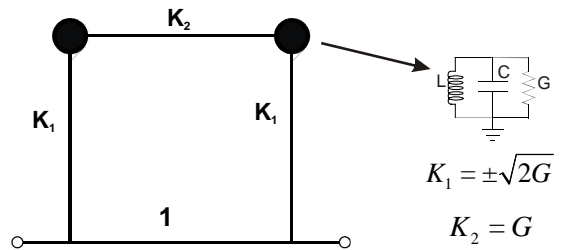


Fig. 4. Generalized coupled-resonator model of a matched notch filter.

Shown in Fig. 4 is a generalized coupled-resonator model of a perfectly-matched notch obtained by scaling nodes of the

admittance matrix of the  $90^\circ$  hybrid circuit in Fig. 2. Several microstrip circuits were designed based on this model, the most practical and successful of which are discussed below. All circuits are fabricated on Rogers Duroid 5880 with a  $\epsilon_r$  of 2.2, a substrate thickness of .787 mm, and a metal thickness of  $24\ \mu\text{m}$ . Circuits were tuned using dielectric overlays and/or selectively removing metallization.

#### A. Twin-Resonator

The design in Fig. 5a consists of two parallel-coupled half-wavelength microstrip resonators coupled to a thru-line. The thru-line is designed to give a  $90^\circ$  phase shift between the resonator couplings. Without tuning the design gave 25 dB of rejection. The center frequency is 956.4 MHz and the 3-dB bandwidth is 12.02 MHz. Recalling that the unloaded  $Q$  of the resonators is simply twice the loaded  $Q$  of the filter,  $Q_u$  is found to be 159.

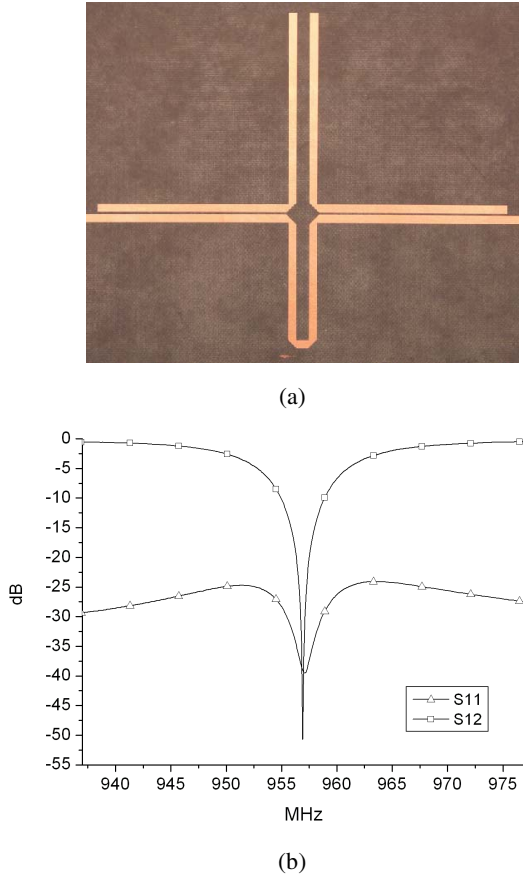


Fig. 5. Twin-resonator notch filter: (a) microstrip prototype ( $120 \times 99\ \text{mm}^2$ ) (b) measured results.

#### B. Dual-Mode

Circuit size can be reduced significantly with the use of dual-mode resonators. Shown in Fig. 6(a,b) are two dual-

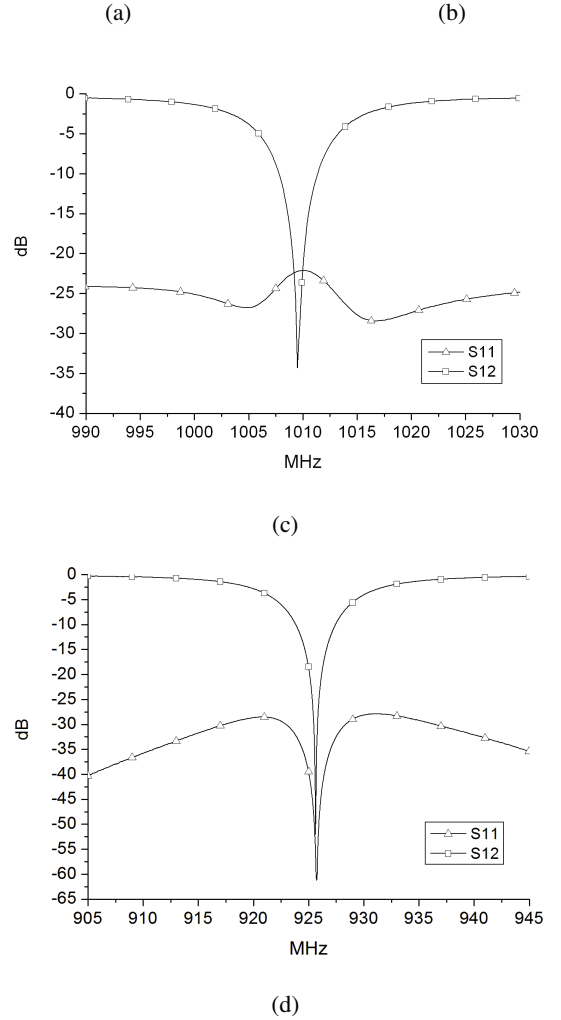
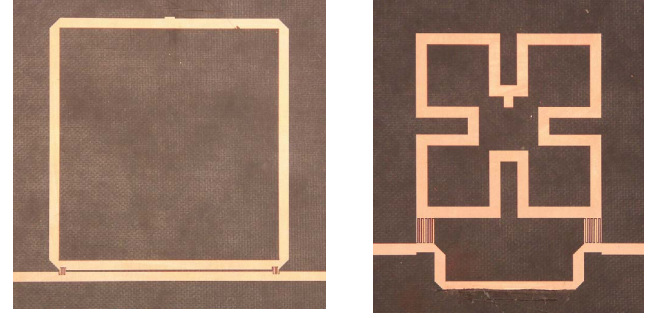


Fig. 6. Dual-mode notch filters: (a) microstrip ring-resonator prototype ( $56 \times 56\ \text{mm}^2$ ) (b) microstrip folded-ring resonator prototype ( $40 \times 40\ \text{mm}^2$ ) (c) ring-resonator measured results (d) folded-ring resonator measured results.

mode notch designs. The design shown in Fig. 6a consists of a ring resonator coupled to a thru-line. Both inductive and capacitive coupling is realized with parallel coupling lines with interdigital capacitors placed  $90^\circ$  apart. The coupling

between the two modes of the resonator is controlled by a small stub located along the line of symmetry. The tuned response is shown in Fig 6c. The center frequency is 1.009 GHz, with a 3-dB bandwidth of 9.6 MHz. The unloaded  $Q$  of the resonator is 208.

A design consisting of a folded ring resonator coupled to a thru-line with interdigital capacitors is shown in Fig. 6b. As in Fig. 6a the coupling between modes is controlled by a tuning stub. The response of the tuned circuit is shown in Fig. 6d. The center frequency is 925.75 MHz and 3-dB bandwidth is 10.9 MHz, giving an unloaded resonator  $Q$  of 168.

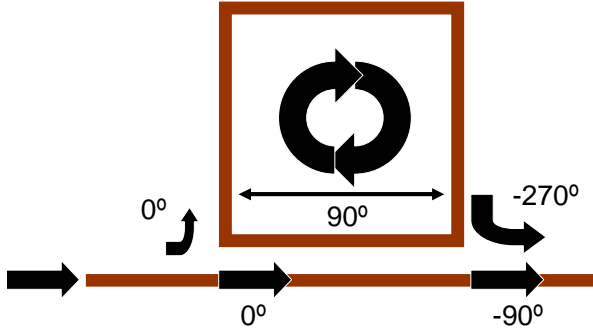


Fig. 7. Traveling-wave interpretation of the dual-mode ring resonator notch. The two modes of the resonator are excited  $90^\circ$  out of phase, resulting in a single circulating wave.

### C. Discussion

The twin-resonator design was found to be the easiest to tune of all the designs due to the lack of interdigital capacitors. The simple ring resonator (Fig. 6a) gave the highest value of  $Q_u$ .

In both dual-mode designs energy is coupled into the two modes  $90^\circ$  out of phase, effectively setting up a single wave circulating around the resonator. This is most obvious in the ring-resonator notch (Fig. 7). At resonance the power coupled off from the resonator at the output is equal in power and  $180^\circ$  out-of-phase with the signal exiting the thru-line, and a perfect notch is produced. When approached from this theoretical

point of view, the capacitors in the prototype are required to compensate for the finite directivity of the coupled lines, which degrades the return loss. The circuit is basically an extension of the traveling-wave ring resonator mentioned in [5].

### IV. CONCLUSIONS/FUTURE WORK

Lossy allpass networks are used to implement perfectly-matched bandstop filters. Relevant theory is presented, along with example designs based on a simple coupled-resonator model. Multi-section matched notch filters are currently being considered, as well as methods to further increase selectivity such as equalization.

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